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Mathematical models for manufacturing a novel gear shaper cutter †

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Abstract

The design principle and models for novel error-free shaper cutters are discussed to improve the accuracy and service life of gear cutting tools. The modified methods for designing the tooth profile of the shaper cutter are developed by including the inverse envelope method, midpoint-midline method and midpoint-displacement method based on the theory of geometrical reverse problem. The universal mathematical models for manufacturing optimal tooth profile of shaper cutters are presented. The feasibility and reliability of the proposed principle, methods and models are proved by combining the numerical examples with practical application. The design method suggested in this paper is superior to the traditional design scheme, and will be helpful in facilitating the design and manufacture of shaper cutters.

Keywords: Mathematical model; Modified tooth profile; Profile error; Shaper cutter

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1. Introduction

Gear shaping is widely used to produce various gears, such as helical gears, external gears and internal gears for distinct features of the manufacturing process. The traditional design scheme on ordinary gear shaper cutters is similar, and a detailed discussion is presented in [1]. Since accuracy and service life of the cutting tool directly affect the manufacturing cost and productivity in gear production, with the increasing demands for high productivity and low manufacturing cost of gear production, the elevation of manufacturing precision and service life of gear shaper cutters have attracted considerable interest. Janninck[2, 3] presented the integrated design method of shaper cutters for various applications. Rogers et al. presented an offset based method to obtain the tooth specifications required for the shaping processes and the strength of the meshing tooth pair[4]. Kim and Kim analyzed the basic theory of pinion cutter shape and developed computer software to design a pinion cutter[5]. Other researchers analyzed the tooth profile undercutting conditions of generating gears by shaper cutters, tooth-profile shifting and gear clearance[6-9]. Tsay *et al* proposed a complete geometrical mathematical model of a spur shaper cutter, including the protuberance, the involute region and semi-topping in manufacturing standard or non-standard spur gears[10]. Bair investigated not only the profiles of the shaper cutter while considering protuberance and semi-topping, but also the relationship between the shaper cutter parameters and the gear tooth profile for the manufacture of helical gears with small numbers of teeth[11]. On the other hand, improvement of tool materials properties was used for increasing service life of tools[12-14]. Some researchers studied the influence of cutting edge form and cutting speeds on tool wear resistance[15] . Kim developed the on-line tool-life monitoring system using acoustic emission signals in gear shaping[16]. In most of these shaper cutter related studies, tooth profile modifications are frequently applied on design and manufacture of shaper cutters for gear accuracy and quality, and materials or cutting condition are investigated on tool life. However, little research work related to the accuracy design with tool service life simultaneously.

It's well known that longer qualified tooth length can improve the service life of a shaper cutter and lower the gear production cost. Tool life is not only determined by the materials and cutting condition, but also has close relationship with the qualified tooth length. A cutting tool can carry on cutting work by repeated sharpening till it is abandoned. In each sharpening, the volume of cutter body is reduced, and a new cutting edge sharpened may have formative error. After the last, oversize error or other reasons cause the cutter be aban-

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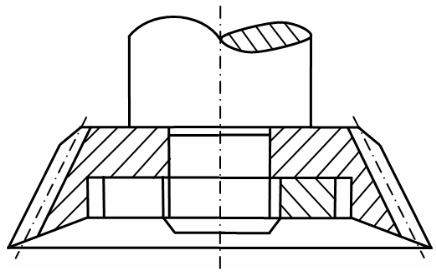
doned.

The qualified tooth length is the length from the cutting edge of a new shaper cutter(called frontal edge for short) to the cutting edge of the final shaper cutter that cannot be used after repeated sharpening(called rear edge for short). The longer qualified tooth length that can be resharpened more times makes the longer tool life. The design accuracy, especially the flank surface of shaper cutter, is beneficial to extend qualified tooth length. However, traditional design approaches for shaper cutter, for example, the involute shaper cutter with module 8, the number of teeth 13, within the tooth length 6mm, the error of cutting edge projection on the end surface to the theoretical involute reaches 0.0133mm[17]. The design error is also large for a non-involute shaper cutter, such as epicycloidal shaper cutter, like the numerical example in [18]; even though the allowable error of tooth top is 0-0.05mm, and the allowable error of the flank is -0.1-0mm, the usable tooth length is only 0.52mm. Therefore, in gear shaping related design, it is important to take both accurate tooth profile and longer qualified tooth length into consideration, which can reduce production cost and increase productivity.

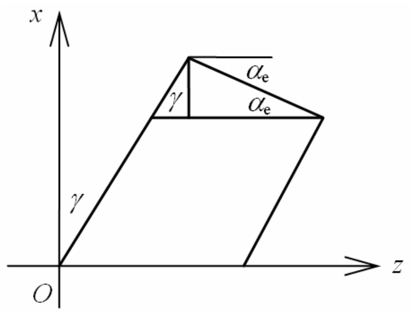
The authors have tried to establish new design methods for novel error-free shaper cutters with longer qualified tooth length; the design principle and general mathematical models are discussed. In this paper, design principle of continuous functions and corresponding mathematical models for error-free novel shaper cutter are introduced firstly. Then based on the gear generation mechanism and the theory of gearing, the reference cross section of the grinding wheel is evaluated by the reversal problem method--reversal envelope method. The absolute value of the tooth profile error of every point on the line from the top to the root is less than 10^{-4}mm, the line is the midline of the qualified tooth length given, so that the reference cross section of the grinding wheel can be modified by midpoint-midline method. According to the error distribution requirement on tooth surface within qualified tooth length, the midpoint of contact line between the grinding wheel and the qualified tooth surface of theoretical shaper cutter should deviate a little in order to make the absolute value of errors on the frontal edge and on the rear edge approximately equal; thus the grinding wheel profile is obtained finally by midpoint-displacement method. This paper presents the universal mathematical models of a new type of design and manufacturing method for different kinds of tooth profiles of shaper cutter; the feasibility and reliability of the principle, methods and models are verified by numerical examples and virtual reality.

2. Design principle and mathematical models for error-free novel shaper cutter

Since the cutting edge clearance of the shaper cutter, tip diameter and tooth thickness of the shaper cutter decrease gradually from front to back, this causes different modification coefficient on different section. Taking involute gears as an example, the profile of the shaper cutter is also involute.



(a) General shaper cutter structure



(b) Rake angle γ and rear angle α_e of shaper cutter

Fig. 1. Diagram of shaper cutter.

Hence rear faces should also be involute helicoid. When rake angle $\gamma = 0°$, the tooth profile of the rake face is involute, so there is no design error. But in general case, $\gamma \neq 0°$, the rake face becomes a cone, the cutting edge is a space curve that is intersected by the rake face and the rear face, and its projection on the end section is no longer involute. This causes tooth profile error, and the error increases with absolute value of rake angle and the rear angle becomes larger. Many researchers have studied gear design and manufacturing with tooth profile shifting. But there still exist tooth undercutting at the generated gear tooth surfaces, under certain conditions, such as small number of teeth, large gear module, big modification coefficient and large absolute value of rake angle γ and rear angle α_e. Especially for hard and brittle characteristics of cemented carbide materials, the rake angle γ of carbide shaper cutter must be negative, of which the absolute value should be greater to prevent tipping. Such shaper cutter causes generated gear flank-concave tooth surfaces. Fig. 1(a) and (b) show diagrams of structure and tool angles(Rake angle γ and rear angle α_e) of shaper cutter, respectively. Fig. 1(b) specially takes carbide shaper cutter with negative rake angle as an example in which z-axis is the axis of shaper cutter and x-axis perpendicular to z-axis. The new design method tries to guarantee projection of every resharpened cutting edge on the end surface coincide with theoretical curve, namely error-free design.

Considering the meshing relation between the blade curve's projection on the end of the shaper cutter and the gear tooth profile, traditional design for the shaper cutter takes the intersectant curve as the blade curve, which the cylinder generated by the conjugate tooth profile of the gear moving around the axis of the gear intersects the rake face of the shaper cutter.

This leads to larger design error. To solve this problem, on the base of gear theory and gear generating mechanism, the developed design principle is that the conjugate curve can be calculated from the conjugate tooth profile of the gear with modification coefficient ξ_1, and that is also the blade curve's projection on end surface of the shaper cutter. For there is rear angle α_e, decrease of tip diameter and tooth thickness of the shaper cutter from front to back causes different modification coefficient ξ on different section. All resharpened cutting edges form the flank surface of the side edge of the shaper cutter farther. So while ξ varies continuously, the flank surface of the side edge will be more accurate than that introduced in [1].

In gear shaping, the velocity of cutting motion is larger than that of generating motion; thus the cylinder generated by the blade curve of the shaper cutter is regarded conjugative with the gear approximately. Therefore, the design principle introduced here is not no-error but it is more accurate than traditional method. The distance from the no-error design is too little to influence the precision of shaper cutter.

Based on kinematics principle, the motion of shaper cutter can be superposed on the generated gear. On any section perpendicular to the gear axis, formative curve of the shaper cutter can be treated as an envelope of gear profile family by continuous movement when section circle of cylinder formed by reciprocating motion of gear pitch circle along shaper cutter does pure rolling movement. Fig. 2 depicts the initial position relationship between the shaper cutter and the gear. Coordinate systems $\sigma=[O; x, y, z]$ and $\sigma_1=[O_1; x_1, y_1, z_1]$ are rigidly attached to the shaper cutter and the gear, respectively. z-axis and z_1-axis are the rotation axes of the shaper cutter and the gear, respectively. Origin points O and O_1 are both on the same plane perpendicular to z-axis. Suppose the gear modification coefficient is ξ_1 in coordinate system σ_1, the transversal curve equation of gear tooth profile on coordinate plane xy can be expressed as follows:

$$\mathbf{r}_1 = \{x_1(\theta), y_1(\theta), 0\} = \{x_1, y_1, 0\} \tag{1}$$

Let the pitch circle with center O_1 roll angle α along the pitch circle of the projecting curve of shaper's blade curve on coordinate plane xy with center O (shown in Fig. 3), and the motion is pure rolling. Hence coordinate system σ_1 has shifted to $\sigma_\alpha=[O_1; x_1, y_1, z_1]$. Eq. (1) can be represented in coordinate system σ as follows:

$$\begin{aligned}\mathbf{r} = \{&x_1\sin(\alpha+\alpha_1) - y_1\cos(\alpha+\alpha_1) + (R+R_1)\cos\alpha,\\ &x_1\cos(\alpha+\alpha_1) - y_1\sin(\alpha+\alpha_1) + (R+R_1)\sin\alpha,\ 0\}\\ =\{&x, y, 0\}\end{aligned} \tag{2}$$

where R and R_1 are the radii of pitch circles with center O and O_1, respectively, and there exists:

$$\alpha_1 = R\alpha / R_1 \tag{3}$$

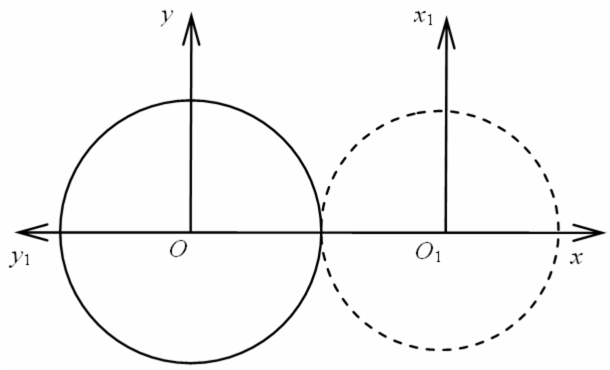


Fig. 2. Initial position of two pitch circles of shaper cutter and generated gear.

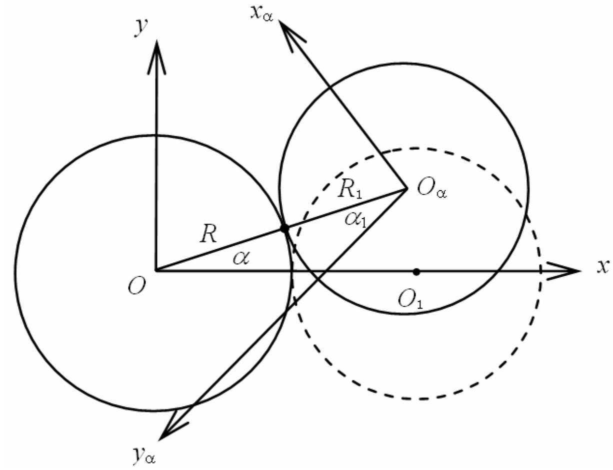


Fig. 3. Relative position of two pitch circles after rolling angle α.

where α and α_1 are angles between line OO_α and x-axis and x_α-axis respectively.

Based on envelope theory, the transversal curve on coordinate plane xy of cylinder generated by the blade curve of the shaper reciprocating in line is the conjugated curve of curve Eq. (1). There exists a formula of envelope condition:

$$\begin{cases} x = x(\theta,\alpha) \\ y = y(\theta,\alpha) \\ z = 0 \\ x_\theta y_\alpha - x_\alpha y_\theta = 0 \end{cases} \tag{4}$$

Then the transversal curve on coordinate plane xy can be deduced:

$$\begin{cases} \mathbf{r} = \{x, y, 0\} \\ x_1 x_1' + y_1 y_1' - R_1(x_1'\sin\alpha_1 + y_1'\cos\alpha_1) = 0 \end{cases} \tag{5}$$

Let $x'/\sqrt{x_1'^2 + y_1'^2} = \sin\varphi$, then the second equation of Eq. (5) can be rewritten as:

$$\alpha = R_1 \arccos((x_1\sin\varphi + y_1\cos\varphi)/R_1)/R + R_1\varphi/R \tag{6}$$

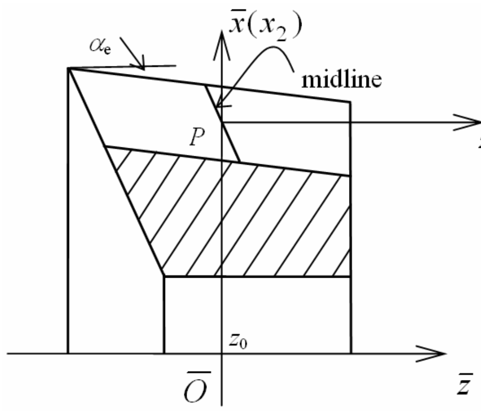


Fig. 4. Cross section of shaper cutter.

where

$$\varphi = \arctan(x_1' \,/\, y_1') \tag{7}$$

Substituting Eq.(7) into Eq.(6), then into Eq.(5), solving Eq.(5) becomes easy, and the corresponding curve equation of the solution can be simplified as

$$\mathbf{r}^* = \{x^*(\theta), y^*(\theta), 0\} \tag{8}$$

Similarly, the corresponding equation of the generated cylinder can be expressed as

$$\hat{\mathbf{r}} = \{x^*(\theta), y^*(\theta), \lambda\} = \{\hat{x}, \hat{y}, \hat{z}\} \tag{9}$$

where λ is a parameter. It should be noticed that Eq. (9) is independent of the coordinate origin. Given the rake angleγ? shown in Fig. 1(b), the equation of the rake face is

$$\begin{aligned}\mathbf{r}_2 &= \{u\cos v, u\sin v, \xi\, m\tan\gamma - \xi\, m\cot\alpha_e\} \\ &= \{x_2, y_2, z_2\}\end{aligned} \tag{10}$$

where α_e denotes rear angle of the shaper cutter like shown in Fig. 1(b), m represents the module of the shaper cutter/gear and coefficient ξ is modification coefficient of the blade curve's projection on end surface of the shaper cutter, that is to say ξ is also the modification coefficient of pitch circle with radius R. u and v are parameters. From Eqs. (9) and (10), the following can be derived:

$$\begin{cases} u = \sqrt{x^{*2}(\theta) + y^{*2}(\theta)} \\ v = \arctan\left(\dfrac{y^*(\theta)}{x^*(\theta)}\right) \end{cases} \tag{11}$$

Notably if ξ is given, the blade curve can be solved from Eqs. (9) and (10) or from Eqs. (10) and (11). While ξ varies continuously, the accurate flank surface of the side edge will be obtained.

3. Reference section of grinding wheel

Usually, gear shaper cutter is ground with a grinding wheel which reciprocates in line quickly and synchronously to do slower generating motion. Since the reciprocation is so quick it can be regarded as the cylinder generated by reciprocating motion of the grinding wheel profile does the generating motion, and the envelope of the family of cylinders is the shaper's backlash flank surface. Once the flank surface of the side edge has been solved, the envelope can be obtained. Reversal evaluation of the cylinder equation in a family of cylinders is an inverse envelope problem. On the other hand, regarding the shaper cutter doing generating motion relative to the grinding wheel, the inverse envelope problem becomes the envelope problem. Furthermore, this problem can be simplified to a 2-D problem as shown in Fig. 4. There exist envelope features between the cylinder generated by the grinding wheel and the transversal of the section perpendicular to z-axis and passing through the midpoint P of the qualified flank surface of the shaper cutter obtained in the section 2. The instantaneous coordinate systems $\bar{\sigma} = [\bar{O}; \bar{x}, \bar{y}, \bar{z}]$ and $\sigma_2 = [O_2; x_2, y_2, z_2]$ are rigidly connected to the shaper cutter and the cylinder generated by the grinding wheel, respectively. Then the following will discuss models of the aforementioned envelope features.

Let coordinates of origin $\bar{O}$ in σ be (0, 0, z_0) obtained in section 3. The equation of the transversal by the flank surface and plane $\overline{xy}$ may be expressed as:

$$\hat{\mathbf{r}} = \{\hat{x}(\xi), \hat{y}(\xi), z_0\} = \{\hat{x}, \hat{y}, \hat{z}\} \tag{12}$$

Hence, the transversal of the cylinder generated by the grinding wheel can be regarded as a tooth profile of a gear rack. According to gear generating mechanism, the equation of the transversal by tooth profile of the rack and the generated cylinder can be obtained.

$$\begin{aligned}\mathbf{r}_3 &= \{\hat{x}\cos\alpha + \hat{y}\sin\alpha, -\hat{x}\sin\alpha + \hat{y}\cos\alpha + R\alpha, z_0\} \\ &= \{x_3, y_3, z_3\}\end{aligned} \tag{13}$$

in which R is pitch radius of the shaper cutter, and α is a parameter that satisfies:

$$\alpha = \arcsin((\hat{x}\sin\varphi + \hat{y}\cos\varphi)/R) - \varphi_1 \tag{14}$$

where

$$\varphi_1 = \arctan((\hat{x}' / \hat{y}') \tag{15}$$

Let the curve of Eq. (13) move along the straight generatrix {sinα_e, 0, -cosα_e}; it will generate the cylinder, then the axial cross section of the grinding wheel generating the cylinder yields:

$$\begin{aligned}\mathbf{r}_s = \{&(\hat{x}\cos\alpha + \hat{y}\sin\alpha - R)\cos^2\alpha_e, \\ &-\hat{x}\sin\alpha + \hat{y}\cos\alpha + R\alpha, \\ &(\hat{x}\cos\alpha + \hat{y}\sin\alpha - R)\cos\alpha_e\sin\alpha_e\} \\ = \{&x_s, y_s, z_s\}\end{aligned} \tag{16}$$

It should be noticed that the cross section of the grinding wheel expressed by Eq. (16) is not the optimal one, and it will cause larger profile errors at the top and root of the gear tooth. So that is called the reference section of the grinding wheel.

4. Actual flank surface and optimization methods for grinding wheel design

The theory of solving the actual flank surface of the side edge should be introduced firstly, such as midpoint-midline point method and center deviation method. Then the corresponding models will be presented.

Notably, according to what was discussed in the section 3, suppose the curve of (13) is a directrix, and $\{sin\alpha_e, 0, -cos\alpha_e\}$ is a straight generatrix; there is generated a cylinder. The actual flank surface is enveloped by the generating motion of the cylinder relative to the shaper cutter.

Midpoint-midline method: the midpoint P of the qualified flank surface of the shaper cutter satisfying the accuracy requirement is taken as the contact point between grinding wheel profile and the shaper cutter. The absolute value of the tooth profile error of every point on the midline from the top to the root on the flank surface is less than10^{-4}mm in order to modify the reference section of the grinding wheel. The absolute values of profile errors of the initial and the last resharpened cutting edge therefore approach to equal and satisfy accuracy demands.

Midpoint deviation method is proposed on the numerical calculating results based on midpoint-midline method and manufacturing reality. The absolute value of profile error on the midline is small, and the absolute error value of the initial and resharpened blade's profile approaches to equal as mentioned above; however, the errors are not equal after all. As long as the midpoint of the contact line between the grinding wheel and the shaper's flank surface shifts moderately towards the absolute value of errors increscent direction, the absolute values of profile error of the whole flank surface will be reduced further.

Based on the aforementioned principle, methods, and combining section 3 of this paper, the corresponding mathematical models are yielded.

In processing, the function of the cross section of the grinding wheel as expression Eq. (16) and the cylinder generated by the directrix expression Eq. (13) moving along straight generatrix $\{\sin\alpha_e, 0, -\cos\alpha_e\}$ is obviously equivalent, and the cylinder's equation can be expressed as

$$\begin{aligned}\mathbf{r}^{**} &= \{\hat{x}\cos\alpha - \hat{y}\sin\alpha - R + \mu\sin\alpha_e,\\ &\quad -\hat{x}\sin\alpha + \hat{y}\cos\alpha + R\alpha, z_0 - \mu\cos\alpha_e\}\\ &= \{x^{**}, y^{**}, z^{**}\}\end{aligned} \tag{17}$$

Let the z-axis coordinate of an arbitrary point on the midline through the midpoint P be $\hat{z}_i$, then the value of parameter μ appearing in Eq. (17) is

$$\mu_i = (z_0 - \hat{z}_i)/\cos\alpha_e \tag{18}$$

Hence, the corresponding point on the generated cylinder by grinding wheel to every point on the midline of the shaper's flank surface can be obtained by substituting Eq. (18) into Eq. (17). Based on the theory of gearing, the coordinates of actual points on the midline of the shaper's flank surface mentioned above are

$$\begin{cases} x_1^* = x_2^* + y_i^{**}(x_i^{**}\cos\varphi_1 - x_2^*)/(R\varphi_1) \\ y_1^* = y_2^* + y_i^{**}(-y_i^{**}\sin\varphi_1 - y_2^*)/(R\varphi_1) \\ z_1^* = z_i^{**} \\ x_{1x^{**}}^* y_{1\varphi_1}^* - x_{1\varphi_1}^* y_{1x^{**}}^* = 0 \end{cases} \tag{19}$$

in which there exist:

$$\begin{cases} x_2^* = (\hat{x}^2 + R^2\varphi_1^2)^{1/2}\cos(\varphi_1 - \arctan(R\varphi_1/\hat{x})) \\ y_2^* = (\hat{x}^2 + R^2\varphi_1^2)^{1/2}\sin(\varphi_1 - \arctan(R\varphi_1/\hat{x})) \end{cases} \tag{20}$$

where R and φ_1 are same as the aforementioned ones, and the fourth equation of Eq. (19) is the envelope condition equation. The midline of qualified flank surface can be derived from Eq. (10)

$$u_i = (\xi m\tan\gamma - \xi m\cot\alpha_e - \hat{z}_i)\tan\gamma \tag{21}$$

Hence, coordinates (x_{2i}, y_{2i}, z_{2i}) of corresponding points on the designed flank surface can be determined. Let

$$x_1^* = x_{2i} \tag{22}$$

Solving simultaneous Eqs. (20) and the fourth equation of (19), x_{1i}^* and φ_{1i} will be obtained, and then substitute them into

$$\Delta y_i = y_{2i} - y_1^* \tag{23}$$

and replace y_i^{**} in Eq.(17) with

$$y_i^{***} = y_i^{**} + \Delta y_i \tag{24}$$

iterative solving from (18) to (22) till

$$\max\{|\Delta y_i|\} < 10^{-4} \tag{25}$$

Suppose ξ_1^* and ξ_2^* are modification coefficients of initial blade and final resharpened blade, respectively, then tooth errors on top and root of the initial blade and final blade may be solved by simulating Eqs. (21), (22) and (23). Furthermore, modified grinding wheel can be obtained by shifting the contact point P on the midline to the direction of maximal absolute value of the tooth error. The procedure is similar to that of the previous section, so the details are omitted here. What should be emphasized is that the constraint of Eq. (25) is

$$\max\left\{\left|\Delta y_i\right|\right\} < 10^{-3} \tag{26}$$

If the constraint can be satisfied, the cross section of the grinding wheel will be the optimal solution. The precision of the qualified tooth surface is the best, and the error distribution tends to be consistent from frontal edge to the rear edge on the whole qualified tooth length.

5. Numerical example and design methods compared

Involute shaper cutters are taken as computational examples here in order to verify the superiority of the proposed design method. Given parameters of shaper cutter, profile angle α_0', base circle radius R_0' and base helix angle β_0' after modification, according to [1], the corresponding formulae are

$$\alpha_0' = \arctan(\tan\alpha_0 / (1 - \tan\alpha_e \tan\alpha_0) \tag{27}$$

$$R_0' = m z_0 \cos\alpha_0' / 2 \tag{28}$$

$$\beta_0' = \arctan(\sin\alpha_0 \tan\alpha_e) \tag{29}$$

In the expressions above, parameters m, α_0, α_e and z_0 represent gear module, original profile angle, rear angle, tooth number, respectively. The equation of flank surface of the side edge is

$$\begin{cases} x^* = R_0'(\cos(\eta - \psi) + \eta\sin(\eta - \psi)) \\ y^* = R_0'(-\sin(\eta - \psi) + \eta\cos(\eta - \psi)) \\ z^* = -\psi R_0 \cot\beta_0' \end{cases} \tag{30}$$

where η is involute parametric variable and ψ is parametric variable of helical rear face. The intersection between two function surfaces of Eqs. (14) and (9) is the cutting edge of the shaper cutter, and while ξ takes different values, different cutting edges can be obtained. The difference between the projection equation of cutting edge on end face and standard involute is a design error of the shaper cutter. On the other hand, the design error of this method introduced by this paper above is zero apparently. Table 1 lists errors of calculation example for traditional design method when m=2, $\alpha_e = 6°$ and $\alpha_0 = 20°$.

It's obvious from Table 1 that a lesser tooth number and larger absolute value of negative rake angle causes larger design error. The following discusses the machining scheme and corresponding models of the novel design method. The following are examples of the error-free design method given by this paper.

Example 1

Simulation for grinding involute carbide shaper cutter with modulus m=2, tooth number 50, rake angle γ= -15^0 and given tooth length 6mm. Table 2 lists calculating results of coordinates of cross-section curve of the grinding wheel, and Table 3 lists the corresponding profile errors of shaper flank surface within tooth length 6mm.

Example 2

Simulation for grinding general involute shaper cutter with γ=5^0, and other parameters is the same as that of Example 1. Table 4 lists the profile errors of shaper flank surface within tooth length 6mm. If the profile error is limited to $\pm 0.007\text{mm}$, it is found that the qualified tooth length can reach 26mm after calculation. So it can be verified that the tool life has been greatly prolonged.

Example 3

The transversal curve equation of designed shaper cutter tooth profile for machining epicycloids gear is:

$$x_1(\theta) = \begin{cases} 2.5\sin\gamma \\ 2.5[21\sin\theta - \sin(21\theta) + \dfrac{\sin(21\theta) - \sin\theta}{\sqrt{2 - 2\cos(20\theta)}}] \end{cases}$$

$$y_1(\theta) = \begin{cases} 50 - 2.5\cos\gamma \\ 2.5[21\cos\theta - \cos(21\theta) - \dfrac{\cos\theta - \cos(21\theta)}{\sqrt{2 - 2\cos(20\theta)}}] \end{cases} \tag{31}$$

$$\gamma \in [0°, 82.60326892°), \theta \in [2.032135°, 9°)$$

As was described in the introduction to this paper, the qualified tooth length is 1.5mm with the design methods and mathematical models of this paper, compared to the traditional design value of the qualified tooth length which is only 0.52mm.

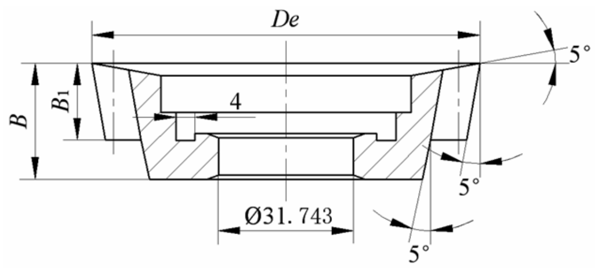


Fig. 5. Parameter diagram of shaper cutter.

Table 1. Tooth profile errors of traditional design method (mm).

		Ordinary shaper cutter		Carbide shaper cutter					
γ		5°		−10°		−15°		−30°	
ξ	z_0	Tooth top	Tooth root	Tooth top	Tooth root	Tooth top	Tooth root	Tooth top	Tooth root
0.315	50	1.75×10^{-3}	1.32×10^{-3}	3.53×10^{-3}	2.68×10^{-3}	5.36×10^{-3}	4.01×10^{-3}	*1.14×10^{-2}	9.02×10^{-3}
	80	1.22×10^{-3}	6.85×10^{-3}	2.45×10^{-3}	1.38×10^{-3}	3.72×10^{-3}	2.11×10^{-3}	7.97×10^{-3}	6.05×10^{-3}
0	50	1.18×10^{-3}	3.10×10^{-3}	2.38×10^{-3}	6.32×10^{-3}	3.01×10^{-3}	9.74×10^{-3}	7.71×10^{-3}	*2.22×10^{-2}
	80	8.10×10^{-4}	1.35×10^{-3}	1.63×10^{-3}	2.75×10^{-3}	2.47×10^{-3}	4.17×10^{-3}	5.30×10^{-3}	9.11×10^{-3}
-0.315	50	7.02×10^{-4}	7.31×10^{-3}	1.41×10^{-3}	*1.34×10^{-2}	2.14×10^{-3}	*1.88×10^{-2}	4.59×10^{-3}	*3.25×10^{-2}
	80	4.73×10^{-4}	2.39×10^{-3}	9.52×10^{-4}	4.84×10^{-3}	1.44×10^{-3}	7.41×10^{-3}	3.10×10^{-3}	*1.63×10^{-2}

Table 2. Coordinates of cross-section curve of the grinding wheel (mm).

x_s	100.00	99.528	99.090	98.679	……	95.667	95.380	95.099
y_s	0.708	0.862	1.003	1.137	……	2.106	2.198	2.288

Table 3. The profile errors of shaper flank surface (mm).

	Top of the tooth	Root of the tooth
Initial blade	-0.0047719	-0.0018780
Midline of blade	-0.0000049	0.0000096
Final blade	0.0045335	0.0016817

Table 4. The profile errors of shaper flank surface (mm).

	Top of the tooth	Root of the tooth
Initial blade	-0.001583	-0.000580
Final blade	0.001498	0.000623

This indicates that the error itself of the traditional design method is larger; the traditional design method is inferior to the error-free design method given by this paper.

The methods have been applied in batch productions. Besides what has been discussed above, this study can also be used in manufacturing other gear shaper cutters, like non-involute shaper and preshaving shaper.

The generating machining is carried on the SYKE9C machining tool made in England. Series products have been developed (some are listed in Table 5), and the corresponding parameters are shown in Fig. 5.

6. Discussion

First, this study presents the general design principle and mathematical models, which can be applied to different kinds of tooth profile design. There only exists a different rake angle γ between ordinary shapers and the carbide shaper. For ordinary shapers the sign of γ is positive and for the carbide shaper, it is negative. As for the involute and non-involute tooth profile, only the function form of Eq. (1) is different. Second, in the introduction of this paper it was emphasized that the qualified tooth length should be as long as possible. But the models discussed in this paper mainly concentrate on satisfying the tooth profile precision. It seems be mutually contradictory. In fact, high design precision can prolong qualified tooth length to a certain extent. From the actual production conditions, the qualified tooth length should be taken into account for the ordinary shaper cutter design; on the other hand, the tooth profile precision should be considered to the carbide shaper because the qualified tooth length has been given already. For the ordinary shaper cutter design, on the basis of precision requirement, namely $|\Delta y_i|$ of Eq. (26) is less than the acceptable error, a longer qualified tooth length conforming with the accuracy requirement will be obtained by change modification coefficients of initial blade and final resharpened blade ξ_1^* and ξ_2^* to lengthen the effective tooth profile reasonably. The mathematical models developed herein can extend the service life of shapers.

Moreover, regarding special required shaper cutters, such as the aforementioned example 3, the tolerance is

$$\Delta y = \begin{cases} \Delta y_i \in [0, 0.05) & \text{top of the tooth} \\ \Delta y_i \in (-0.1, 0) & \text{root of the tooth} \end{cases} \tag{32}$$

The tolerance is bigger than that of other kinds of tooth profile shaper cutter.

7. Conclusions

Because of the limited length, this paper only gives the main models and omits the details. The details can be easily derived from the methods mentioned above. On the basis of the idea, the principle, mathematical models and numerical examples mentioned in this study, and practical production in Harbin No.1 Tools Factory, the conclusions are generalized as follows:

(1) The novel design scheme is error-free. It is obviously better than the traditional design scheme, especially for design carbide shaper cutters with large module and large negative rake angle or special profile shaper cutters.

(2) The novel error-free design scheme and the proposed methods and mathematical models in this paper can produce high-accuracy carbide shaper cutters and

Table 5. Products table of BC_2 series.

Product serial number	Tooth number	De (mm)	B (mm)	B1 (mm)	Short width coefficient	Eccentricity (mm)
BC_2-001	25	78	32	25	1	1.5
-002	24	100		26		2
-003	20	105				2.5
-004	18	114				3
-005	16	119				3.5
-006		120				4
-007	13	126	35	29		4.5
-008	12	130				5
-009	11	132	36	31		5.5
-010	10					6

greatly increase qualified tooth length of general shaper cutters.

(3) The design scheme, principle and mathematical models presented in this study have generality, which can be used to design and produce shape cutters of various shapes.

(4) Based on the proposed design principle, the reversal envelope method for solving the cross section of grinding wheel, midpoint-midline method and midpoint-displacement method not only can be used to improve the tooth profile precision of shaper cutters but also be applied in complex cutting tools design and manufacture, such as broaches and hobs for high accuracy and long service life of the tools. Hence, this paper has reference value not only to shaper cutters, but to other complex cutting tools as well.

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